Optical Considerations for the Long Trace Profiler: Required Camera Resolution

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Abstract

As requirements for measuring mirrors with more accuracy become more demanding, knowledge of the Long Trace Profiler's (LTP's) ability to measure small slope differences between adjacent locations on a mirror becomes more important. The kind of slope resolution one can expect of a measurement, given the optical system parameters and slope conversion algorithm, is presented.

Consideration of the optical system parameters and especially the type of algorithm that is used, makes an analytical determination of slope resolution difficult. However, computer simulation of intensity pattern generation and analysis allows quantitative slope resolution estimates to be made for any optical system. These estimates are compared with actual measurements.

Introduction

Since the Long Trace Profiler was first built a decade ago[1] there was the question, "What kind of camera is needed for acquiring intensity patterns that are created by the interference of the probe beam pair?" Parameters of a camera that must be considered include physical size, interface to computer, data transmission speed, number of bits per pixel, number of pixels per array, array size, dark current, and responsivity to red light. Mechanical and geometrical optics issues immediately determined all of these parameters except for the dynamic intensity resolution (number of bits per pixel). It was generally understood that the camera needed enough resolution so that the intensity pattern processing algorithm could distinguish between small fractions of a microradian (µrad); other sources of error would likely be significant just under 1 µrad.

The dynamic resolution was determined experimentally circa 1990 by observing the LTP performance with cameras available at the time. The vague determination was that 8 bits per pixel was certainly not good enough, and 12 bits was marginally acceptable. Since that time the LTP has been improved by compensating for repeatable errors and reducing unrepeatable errors[2,3], in accordance with increasing demand for improved measurement accuracy. Also, investigation into possible sagittal with tangential profiling (3D) has given an incentive for understanding camera requirements. Thus, a more detailed treatment of required resolution is desired.

A slope change of the surface under test (SUT) produces a change in the intensity pattern at the camera which is recorded in the computer. Measuring a mirror requires determining the slope change of the SUT, and this is inferred by the change of the intensity pattern, which is a translation of the pattern along the camera array. Actually, the entire LTP optical system determines the smallest detectable slope change that can be measured, and this can be explained by geometrical optics. Given the focal length f, detector array length L, and number of pixels in the array N, the smallest measurable slope difference Δs will be proportional to L / (f N):

$$\Delta s \;=\; \frac{K \quad L}{f \quad N} \,. \tag{1}$$

K is a number that makes Δs much smaller, and depends on the camera dynamic resolution and algorithm used for converting intensity pattern shifts to slope values. Two models for how a SUT slope change modifies the intensity pattern have been considered in the past. One model[4] assumes that the intensity pattern is approximately a parabola (true for a few pixels in the center of the parabola). The other model[5] assumes that the intensity pattern is a sinusoid pattern modulated by a gaussian such that about one period of the sinusoid is visible.

The algorithm from the first model selects a few points (pixels) at the center of the pattern and determines a best-fit parabola to these intensity values. The position of the minimum value of the parabola is the effective pattern position, and a change in this position is proportional to the slope change. This is called the curve fit algorithm.

The algorithm for the second model takes the Fourier transform of the intensity pattern with respect to the entire camera array, and determines the phase value in the neighborhood of the sinusoid frequency. The phase value cannot be the simple result of taking the arctan of (imag/real), which will yield numerous discontinuities, but should be an unwrapped phase value that is linearly increasing over the length of the array. This is essentially working the problem backwards, calculating what phase difference within the probe beam pair must have caused such a sinusoid shift. This is called the Fourier transform algorithm.

For either of the two algorithms, the goal is to obtain the greatest slope resolution, which means obtaining the greatest effective detector resolution. The effective detector resolution depends on the number of pixels that cover the slope range of the LTP, to be sure, but also on the intensity resolution (number of bits per pixel) and the computer algorithm that interpolates the pattern position with sub-pixel precision. This paper intends to determine the available slope resolution as a function of intensity resolution and computer algorithm.

In order to determine the effect of camera resolution on the ability to detect a small change in slope, a Matlab™ program was written to artificially create intensity patterns. Each pattern was dithered along the detector array axis by adding one (1) to the intensity values on one side of the pattern. (Each intensity value is an integer that is between 0 and at most (2^nb - 1), where nb is the dynamic resolution [bits per pixel].) This has the effect of shifting the pattern the smallest amount, while making sure that an effect can be seen when the curve fit algorithm is used for any number of fitting points. (See Figure 1.) In each case the detector length L was 25 mm, the focal length f was 1250 mm, and the number of pixels N was 1024. Then K = 51200 Δs.

Finally, measurements were made of actual slope changes. These measurements are compared to what would be expected for various algorithms and dynamic resolutions.

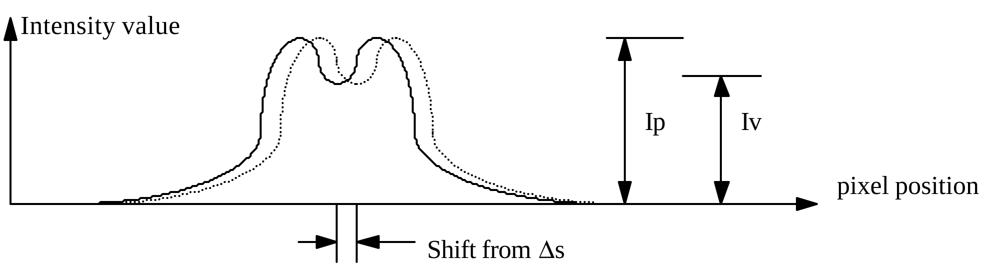

Figure 1. An intensity pattern.

Analytical predictions

The table below gives the values of Δs [μrad] for various values of Nfp (number of fitting points in the curve fit algorithm), nb (camera dynamic resolution), and C (intensity pattern contrast). The intensity pattern contrast is the visibility of the sinusoidal component of the intensity pattern, and is calculated as

$$C = \frac{Ip - Iv}{Ip}, \tag{2}$$

where Ip is the peak intensity value and Iv is the intensity value at the sinusoid minimum shown in Figure 1.

The table may be easier understood by looking at the plot of these values in Figure 2.

Algorithm	nb = 8	nb = 12	nb = 16
C = 0.1			
Curve fit, Nfp = 3	2.093	0.1144	0.0071
Curve fit, Nfp = 7	1.094	0.0688	0.0043
Curve fit, Nfp = 11	1.237	0.0770	0.0048
Fourier transform	0.0715	0.0045	0.00028
C = 0.9			
Curve fit, Nfp = 3	0.3889	0.0239	0.0015
Curve fit, Nfp = 7	0.2116	0.0132	0.00082
Curve fit, Nfp = 11	0.1855	0.0116	0.00072
Fourier transform	0.0665	0.0042	0.00026

Table 1. Expected slope resolution values [microradians].

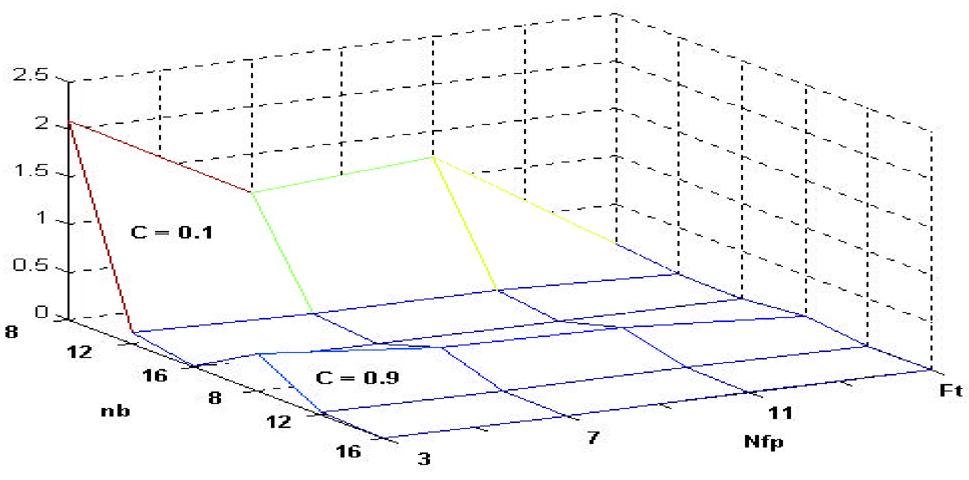

Figure 2. A plot of Δs values from Table 1. Two sets of Δs are given on the nb axis for different values of contrast, C = 0.1 and C = 0.9.

As mentioned above, these figures were made by first generating an intensity pattern using a Matlab™ program, then generating a similar intensity pattern with the smallest effective slope change. Instead of shifting the pattern in the detector pixel direction, which would show a very large slope change, the pattern was shifted in intensity value incrementally for just one side of the pattern. The set of patterns was then processed using the available algorithms already implemented in the LTP control program for Windows™ (LTPw). An even smaller effective slope change could be generated by shifting the intensity values of only a few pixels on one side of the pattern. Which pixels would be shifted? The Fourier transform algorithm will detect a change for any number of shifted pixels, but the curve fit algorithm will detect a change only for those fitting points that coincide with the shifted pixels. For this reason, an unbiased situation was selected for all cases (Nfp = 3, 7, 11, and Ft). Keep in mind that the actual smallest detectable slope changes would be somewhat smaller than shown in this report.

Comparison to measurements

Two measurements were made of a mirror at one place on the mirror (stability scan in LTPw). In the first measurement the mirror was tilted a known amount between pictures. (A picture is a single camera acquisition which records all intensity patterns, and would be a record of slope at one position on the mirror if a measurement scan were performed. The definition of 'picture' is given in a memo[6] of LTP scan conventions.) This amount was set mechanically to about 18 μrad, but the uncertainty of this setting was much greater than any measurement error. In the second measurement there was no intentional tilting of the mirror between pictures. Environmental effects gave the smallest typical change of slope between pictures, which was in the order of 0.2 μrad.

The slope differences between two sample points were recorded and are shown in Tables 2 and 3. Table 2 shows these values for the first measurement and Table 3 shows these values for the second measurement. Vrms is the root mean square variation of Δs as the dynamic resolution changes for a given algorithm and Nfp. The measurement pictures for the lower resolutions (nb = 8, 12) were obtained by quantizing the 16 bit pictures with fewer bins.

The values of Δs from the Fourier transform algorithm are consistently higher than from the curve fit algorithm for the nominal Δs of 18 μrad, while values of Δs from the Fourier transform algorithm are consistently lower than from the curve fit algorithm for the nominal Δs of 0.2 μrad. At this time there is no way of knowing which values are closer to true values.

Nfp	nb = 8	nb = 12	nb = 16	Vrms
3	18.823	18.693	18.710	0.0577
7	18.154	18.220	18.221	0.0313
11	16.877	16.909	16.910	0.0153
Ft	19.547	19.559	19.558	0.0054

Table 2. Measured Δs for a nominal Δs of 18 μrad.

Nfp	nb = 8	nb = 12	nb = 16	Vrms
3	-0.1996	-0.1871	-0.1686	0.0127
7	0.2535	0.2274	0.2289	0.0120
11	0.1379	0.2055	0.2079	0.0324
Ft	0.1102	0.1038	0.1047	0.0028

Table 3. Measured Δs for a nominal Δs of 0.2 μrad.

We might expect $\Delta s = 0$ when the slope resolution is greater than the actual slope change. Instead, the algorithm usually gives some indication of a slope change. Notice that for small actual slope changes, the computed Δs can be much larger or negative compared to the more believable value, as is shown in Table 3.

Caveat on the Fourier transform algorithm

The Fourier transform algorithm appears from this report to be superior to the curve fit algorithm. The Fourier transform method is also desirable if the intensity patterns are not symmetric or if the sinusoid peak coincides with the gaussian peak. However, there are reasons why the curve fit algorithm continues to be used more frequently, other than the fact that LTP control programs that most operators have only contain the curve fit algorithm. First, the Fourier transform algorithm takes two to three times more time for processing the data, although with the speed of today's computers this may not be objectionable. Second, the Fourier transform algorithm gives slightly different surface shapes for a given measurement, as is evidenced by the Δs values in Table 2. These differences are usually noticed at the high to mid spatial frequencies. However, these small differences are often overwhelmed by other errors in an LTP measurement.

It is known that as the intensity pattern moves along the detector axis when a high curvature mirror is measured, the gaussian component does not move at the same rate as the sinusoidal component. This is one of the manifestations of optical system nonlinearity. Unfortunately, the sinusoidal component seems to be affected more by this nonlinearity than the gaussian component. The Fourier transform algorithm is very sensitive to the sinusoidal phase changes, and displays small amplitude periodic variations in the slope function that the curve fit algorithm does not detect. Until this phenomenon is understood and the Fourier transform algorithm is compensated for this sensitivity, this algorithm should be used with caution.

Conclusion

Guidelines on achievable slope resolution have been given, although it is understood that the values for Δs may be somewhat smaller than those shown in Table 2. Measurements substantiate the expected values, but do not give identical results.

The Fourier transform algorithm appears here to be superior to the curve fit algorithm. However, at this time the Fourier transform algorithm may introduce small amplitude periodic artifacts when measuring mirrors with significant curvature.

Acknowledgments

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